

## LARGE EDDY SIMULATION OF NON-ISOTHERMAL FLOW IN ROTOR/STATOR CAVITY

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The paper presents the 3D LES study of the non-isothermal transitional and turbulent flow in rotor/stator sealed cavity. Computations have been performed for the cavity of aspect ratio  $L=5$ , curvature parameter  $R_m=1.8, 3.0$  and  $5.0$  and for the thermal Rossby numbers up to  $0.2$ . Computations we based on the efficient pseudo-spectral Chebyshev-Fourier method. In Large Eddy Simulations we used a version of the dynamic Smagorinsky eddy viscosity model proposed by Meneveau [1996], in which the averaging is performed over the fluid particle pathline.

### INTRODUCTION

The present study concerns the numerical prediction of the transitional and turbulent flows with heat transfer in an enclosed rotor/stator cavity. The problem is not only very interesting from the point of view of fundamental fluid mechanics but it is also a topic of practical importance. The flow in the rotating cavity is of great interest for the internal aerodynamics of engines, especially for the optimization of turbomachinery air-cooling devices. The experimental investigations in the rotating cavities are very difficult and expensive. In this situation, numerical simulations, particularly Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES), which can deliver precise knowledge on the flow structure and temperature distributions in the rotating cavity become indispensable tools. Numerical modeling of the flow in the rotor/stator cavity turned out to be a difficult problem mostly due to the fact that in the cavity simultaneously exist areas of laminar, transitional and turbulent flow which are completely different in terms of flow properties. The present paper is devoted to a study of the turbulent and transitional flow with heat transfer in sealed rotor/stator cavities of aspect ratios  $5$  and curvature parameters  $1.8, 3.0$  and  $5.0$ . The cavity is heated from below and from the outer end-wall, whereas the rotor and the inner cylinder are cooled (Fig.1). The main motivation of our work is to analyze the properties of turbulence of the non-isothermal flow dominated by Coriolis and centrifugal forces.

### NUMERICAL MODELING

The geometrical domain is presented in Fig.1. The upper disk rotates at a uniform angular velocity  $\Omega$  around the central axis. The outer cylinder of radius  $R_1$  and inner cylinder of radius  $R_0$  are attached to the stator and rotor, respectively. The interdisk spacing is denoted by  $2h$ . The flow is controlled by the following physical parameters: the Reynolds number, based on the external radius of the disks  $R_1$  and on the angular velocity of the rotor,  $Re = \Omega R_1^2 / \nu$ , the aspect ratio  $L = (R_1 - R_0) / 2h$ , the curvature parameter  $R_m = (R_1 + R_0) / (R_1 - R_0)$ , the Prandtl number  $Pr = 0.71$  and the thermal Rossby number  $B_r = \beta(T_2 - T_1)$  (where  $\beta$  is the thermal expansion coefficient,  $T_1$  is the temperature of the upper

rotating disk and the inner cylinder, and  $T_2$  indicates the temperature of the stator and the outer cylinder). The flow is described by the continuity, Navier-Stokes and energy equations. The equations are written in a cylindrical coordinate system  $(r, \varphi, z)$ , with respect to a rotating frame of reference. To take into account the buoyancy effects induced by the involved body forces, the Boussinesq approximation is used.

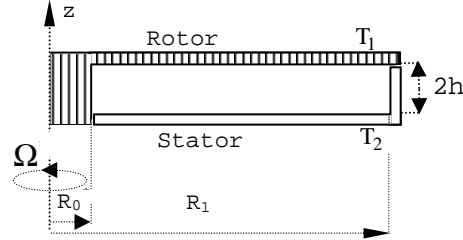


Figure 1. Schematic picture of the rotor/stator cavity.

The numerical code prepared in the present research for LES of the non-isothermal flow in the annular cavity [Tuliszka-Sznitko et al. 2009] is an extended version of the DNS algorithm [Serre and Pulicani 2001]. The numerical solution is based on a pseudo-spectral Chebyshev-Fourier-Galerkin approximation and the time scheme is semi-implicit and second-order accurate (it corresponds to a combination of the second-order backward differentiation formula for viscous diffusion terms and the Adams-Bashforth scheme for non-linear terms). In the LES, after a filtering operation has been applied to the governing equations, with filter width equal to the grid spacing in azimuthal direction, we obtained the filtered equations of motion [Tuliszka-Sznitko et al. 2009]. Subgrid-scale stress tensor  $\sigma_{ij}^{SGS}$  and energy flux  $\alpha_j^{SGS}$  are expressed as:

$$\sigma_{ij}^{SGS} = -2\nu_{SGS}\bar{S}_{ij}, \quad \alpha_j^{SGS} = \frac{\nu_{SGS}}{Pr_{SGS}} \frac{\partial \zeta^k}{\partial x_j} \frac{\partial \bar{T}}{\partial \zeta^k} \quad (1a)$$

$$\bar{S}_{ij} = \frac{1}{2} \left( \frac{\partial \zeta^k}{\partial x_j} \frac{\partial \bar{u}_i}{\partial \zeta^k} + \frac{\partial \zeta^k}{\partial x_i} \frac{\partial \bar{u}_j}{\partial \zeta^k} \right), \quad \nu = C_S^2 \Delta^2 \sqrt{2\bar{S}_{ij}\bar{S}_{ij}} \quad (1b)$$

where filtered dependent variables are indicated by overbars. In the above equations  $(x_1, x_2, x_3) = (x, y, z)$  and  $(\zeta^1, \zeta^2, \zeta^3) = (r, \varphi, z)$ . In our computations, we used a version of the dynamic Smagorinsky eddy viscosity model proposed by Meneveau et al. [1996]. Meneveau et al. [1996] accumulated the required in LES averaging over the fluid particle pathlines, instead of averaging over the direction of statistical homogeneity. In this approach the Smagorinsky coefficient, at a given position  $x$ , depends on the history of the flow along the pathline. The Smagorinsky coefficient is determined by minimizing the modeling error over the pathline of the fluid particle.

## RESULTS

For all considered Reynolds numbers  $25000 \leq Re \leq 350000$  the flow exhibits typical Bachelor behaviour i.e. the flow consists of two disjoint boundary layers on each disk and of a central inviscid core flow. The flow is pumped radially outward along the rotor and recirculates along the stator. In the transitional boundary layers the axisymmetric propagating vortices interpreted as the type II instability and positive spiral vortices interpreted as the type I instability were observed. For

higher Re structures evolve from the spiral vortices to more annular vortices. Fig.2 shows the iso-lines of the azimuthal velocity component disturbances in the azimuthal sections of the stator boundary layer ( $L=5$ ,  $Rm=3$ ,  $Re=200000$ ,  $B=0.1$ ). Computations have shown that the turbulence is mostly concentrated in the stator boundary layer with a maximum at the junction between the stator and outer cylinder. In Figs.3a and b we analyzed the axial profiles of two main Reynolds stress tensor components: azimuthal  $\sqrt{v'v'}(Rm+1)/(Rm+r)$  and radial  $\sqrt{u'u'}(Rm+1)/(Rm+r)$  (profiles were obtained in the middle section of the cavity  $L=5$ ,  $Rm=1.8$ ,  $Re=100000$ ,  $B=0.1$ ). Results are compared with the experimental and numerical data of Séverac et al. [2007].

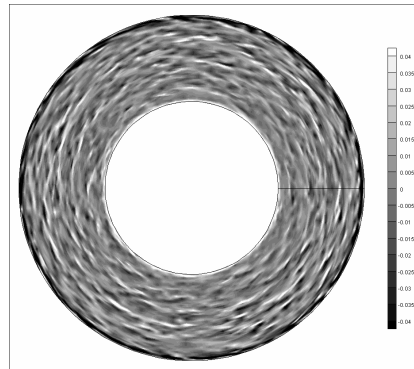


Figure 2. Iso-lines of the azimuthal velocity component; azimuthal section of the stator boundary layer. ( $L=5$ ,  $Rm=3$ ,  $B=0.1$ ,  $Re=20000$ ).

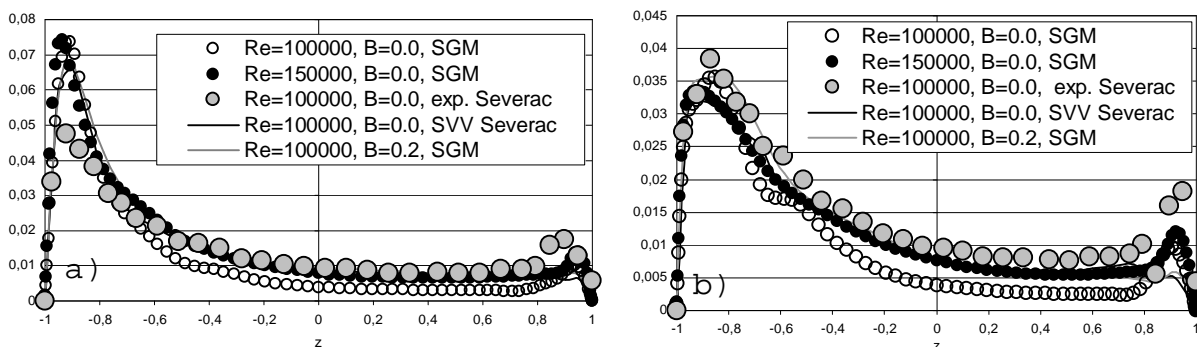


Figure 3. Axial profiles of the azimuthal and radial Reynolds stress tensor components: a) azimuthal  $\sqrt{v'v'}(Rm+1)/(Rm+r)$ , b) radial  $\sqrt{u'u'}(Rm+1)/(Rm+r)$ . ( $L=5$ ,  $Rm=1.8$ ).

One of the most important information from the engineering point of view is the distribution of the local Nusselt number in function of the radius of the disk. Fig.4 shows exemplary distributions of the local Nusselt number  $Nu_r$  along the stator obtained for different Re, ( $L=5$ ,  $Rm=3$ ,  $B=0.1$ ). Based on our computations we proposed correlations formulas for the local an averaged Nusselt number. We analyzed distributions of the turbulent Prandtl number  $Pr_t = (-\overline{v'w'})/\overline{\partial v/\partial z} / (\overline{\Theta'w'})/\overline{\partial \Theta/\partial z}$  and compared the axial profiles with these obtained by Elkins and Eaton [2000] for single heated rotating disk.

## CONCLUSION

In the paper we presented Large Eddy Simulation of the transitional and turbulent non-isothermal flow in sealed cavities of aspect ratio  $L=5$ . In the LES we used a version of the dynamic

Smagorinsky eddy viscosity model proposed by Meneveau [1996], in which the Smagorinsky coefficients are averaged along fluid pathlines. This algorithm turned out to be very effective and allowed us to perform computations for high Reynolds numbers.

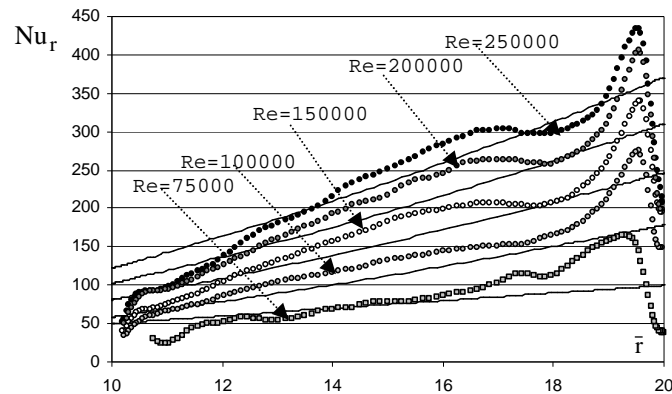


Figure 4. Distributions of the local Nusselt numbers along heated stator. ( $L=5$ ,  $Rm=3$ ).

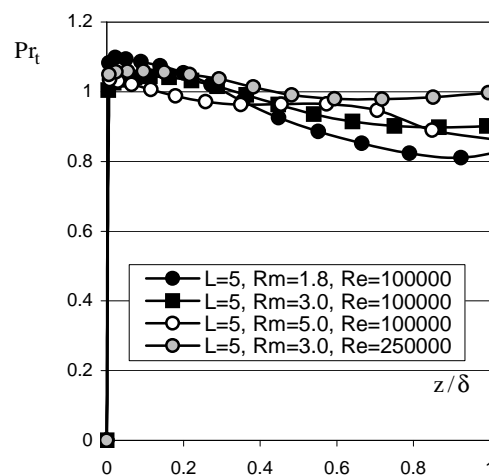


Figure 5. Axial distributions of the turbulent Prandtl numbers  $Pr_t = (-\overline{v'w'}/\partial v/\partial z)/(\overline{\Theta'w'}/\partial\Theta/\partial z)$  obtained in the middle section of cavities.

## REFERENCES

- Elkins, C.J., Eaton, J. [2000], Turbulent heat and momentum transport on a rotating disk, *J. Fluid Mech.*, Vol. 402, pp. 225-253.
- Meneveau, C., Lund, T.S., Cabot, W.H. [1996], A Lagrangian dynamic subgrid-scale model of turbulence, *J. Fluid Mech.*, Vol. 319, pp. 353-385.
- Serre, E., Pulicani, J.P. [2001], A three-dimensional pseudospectral method for rotating flows in a cylinder, *Computers & Fluids*, 30, Vol. 491.
- E. Séverac, E., Poncet, S., Serre, E. [2007], Large eddy simulations and measurements of turbulent enclosed rotor-stator flows, *Phys. Fluids*, 19, 085113.
- Tuliszka-Sznitko, E., Zielinski, A., Majchrowski, W. [2009], LES and DNS of the non-isothermal transitional flow in rotating cavity, *Int. J. of Heat and Fluid Flow*, doi: 10.1016/j.ijheatfluidflow.2009.02.010.