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SIMPLE APPROACH TO MODELING HEAT TRANSFER DURING SOLAR HEATING AND MELTING OF LAKE OR SEA ICE

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ABSTRACT Solar heating and ice melting on the water surface is an important geophysical problem that has attracted the attention of researchers for many years. It is essential in connection with global climate change on our planet. A simple and sufficiently accurate physical model of the process is proposed, combining analytical solutions for the solar radiation transfer in light-scattering snow cover and ice layer with numerical calculations of transient heat transfer in a multilayer system. The boundary conditions for the heat transfer problem consider convective heat losses to the cold air and radiative cooling of the open surface in the mid-infrared window of transparency of the cloudless atmosphere. Much attention is paid to modeling the anomalous spring melting of ice covering the large high-mountain lakes of Tibet, the Earth's third pole. It was found that a thick ice layer not covered with snow starts to melt at the ice-water interface due to volumetric solar heating of ice. The results of the calculations are in good agreement with the field observations. The computational analysis showed a dramatic change in the process when the ice surface is covered with snow. A qualitative change in the physical picture of the process occurs when the snow cover thickness increases to 20–30 cm. In this case, the snow melting precedes ice melting and water ponds are formed on the ice surface. This is a typical situation for the Arctic Sea ice cover during the polar summer. Known experimental data are used to estimate the melting of sea ice under the melt pond. Positive or negative feedback related to the specific optical and thermal properties of snow, ice, and water are discussed.

KEYWORDS: Heat transfer; Solar radiation; Ice; Snow; Scattering; Melting; Lake; Sea

NOMENCLATURE

a	radius of ice grain or gas bubble
c	specific heat capacity
d	geometrical thickness
E	exponential function
f	fraction
f_1, f_2	functions introduced by Eq. (16b)
G	irradiation
H	thickness of water layer
h	heat transfer coefficient
I	radiation intensity
J	diffuse radiation intensity
K	coefficient in Eq. (30)
k	thermal conductivity
L	latent heat of melting
n	index of refraction
P, p	absorbed radiation power
Q, q	heat flux
R	reflectance
S	scattering parameter
T	temperature
t	time
x	diffraction parameter
z	vertical coordinate

Greek symbols

α	absorption coefficient
β	extinction coefficient
ε	coefficient in Eq. (22c)
ζ	coefficient introduced by Eq. (8a)
η	coefficient in Eq. (15)
θ	zenith angle
κ	index of absorption
λ	wavelength
μ	cosine of an angle
ν	parameter in Eqs. (8a-c)
ξ	parameter in Eq. (7a)
ρ	density
σ	scattering coefficient
τ	optical thickness
φ	coefficient in Eq. (12)
ψ	coefficient introduced by Eq. (8b)
ω	albedo of single scattering

Subscripts and superscripts

air	air
b	blackbody
cond	conductive
conv	convective
d-h	directional-hemispherical
day	day
dif	diffuse
dl	daylight
i, inc	incident
ice	ice
init	initial
inf	infrared
j	refracted
loss	loss
max	maximum
n-h	normal-hemispherical
op	opacity
p	pond
rc	radiative cooling
sol	solar
surf	surface
th	thermal
tr	transport
uv-vis	ultraviolet-visible
v	volume
w	water, window
λ	spectral
*	critical

INTRODUCTION

The paper presents a general approach to solving geophysical problems related to solar heating and ice melting on the water surface. The spring melting of lake ice is a relatively simple problem and this will be considered in more detail. After that, a general view of the more complex problem of partial or complete melting of sea ice during the polar summer will be discussed. It is known that ice melting on the Arctic Sea surface is very important for global climate change. What is less well known is that the opening from the ice of high-mountain lakes in Tibet, sometimes referred to as the Earth's third pole [Pan et al. 2021], significantly affects the climate of central Asia. Recall that the Tibetan Plateau is home to about a thousand large lakes with a total area of about 15,000 km^2 [Su et al. 2020, Zhang and Duan 2021]. Interestingly, the opening time of these lakes is an indicator of global climate change [Su et al. 2019, Zhang and Duan 2021].

It should be noted that the physical picture of heating and melting of the ice cover on the lake surface turns out to be quite different for the case when the ice is not covered with snow and in the case when a snow cover is present on the ice surface. However, the approach to solving problems of radiative transfer in snow and ice layers has a common methodological basis, and this can be used in theoretical studies of both lake and sea ice melting. In the spectral range of semitransparency of snow and ice, one should take into account the scattering of radiation either by ice grains in snow or by gas bubbles, which are usually contained in the ice. This means that one should focus on choosing a simple and sufficiently accurate method for solving the radiative transfer equation (RTE) [Dombrovsky and Baillis 2010, Howell et al. 2021, Modest and Mazumder 2021].

The main difficulties in radiative transfer modeling are caused by the RTE's integral term, which takes into account the anisotropic scattering of radiation by particles or bubbles in the medium and contains a scattering phase function. Fortunately, we deal with multiple scattering of radiation in an optically thick medium and the transport approximation is usually quite sufficient. According to this approach, the scattering function is replaced by a sum of the isotropic and forward components. The resulting transport RTE looks like that for the hypothetic isotropic scattering but with the transport scattering coefficient [Dombrovsky 1996a, 2012, 2019]. The transport approximation has been successfully employed in diverse applied problems of radiative transfer in thermal engineering [Dombrovsky 1996a,b, 2001, Dombrovsky et al. 2009a,b, 2017, 2020], biomedicine [Tuchin 2007, Sandell and Zhu 2011, Dombrovsky et al. 2012, 2015, Jacques 2013, Eisel et al. 2018, Dombrovsky 2022], and geophysics [Dombrovsky and Kokhanovsky 2020-2023, Dombrovsky et al. 2019, 2022].

Another significant simplification of the problem is possible due to the linearity of the RTE. In the case of direct solar irradiation of a scattering medium, the radiation intensity can be represented in the form of two additive components: direct radiation and a diffuse component formed due to the scattering. In highly scattering media, the diffuse component of radiation intensity can be calculated using one of the simplest differential approximations: either the known P_1-approximation of the spherical harmonics method (Eddington approximation) or the two-flux approximation (Schuster–Schwarzschild approximation). Note that for refracting media, such as ice, a modified version of the two-flux approximation suggested by Dombrovsky et al. [2006], which takes into account the effect of total internal reflection, can be used. The differential approximations make it possible to replace the RTE by a boundary-value problem for a second-order ordinary differential equation for the irradiation function. The choice between these approximations is determined by the problem statement. Comparison with exact numerical solutions [Dombrovsky and Baillis 2010, Dombrovsky et al. 2013] has shown that the two-flux method is preferable when solving one-dimensional problems typical of solar heating. In this case, the error of the ordinary diffusion approximation (P_1) is larger because this method does not take into account the discontinuity of the angular dependence of the radiation intensity on the illuminated surface of the medium.

TRANSIENT HEAT TRANSFER MODEL

There are various thermal processes in a snowpack or scattering ice sheet and these processes should be involved in a complete heat transfer model. One can recall the ice sublimation and diffusion of water vapor through a snow layer. This may be important for the snow microstructure [Kaempfer and Plapp 2009, Pinzer et al. 2012]. However, the related effects are expected to be not significant in the problem under consideration. The 1D transient energy equation in a layer of snow or ice and the accompanying initial and boundary conditions can be written as follows:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + P, \qquad t > 0, \quad 0 < z < d_{\mathrm{th}} \tag{1a}$$

$$T(0, z) = T_{\mathrm{init}}(z) \tag{1b}$$

$$z = 0, \;\; k \frac{\partial T}{\partial z} = h(T_{\mathrm{air}} - T) - q_{\mathrm{rc}}, \qquad z = d_{\mathrm{th}}, \;\; \frac{\partial T}{\partial z} = 0, \tag{1c}$$

where $T(t, z)$ is the temperature, t is the current time, z is the is the coordinate measured from the illuminated surface, ρ, c, and k are the density, the specific heat capacity, and the thermal conductivity of the medium, $T_{\text{air}}(t)$ and $h(t)$ are the temperature of ambient air and convective heat transfer coefficient which depends on the wind velocity. The adiabatic condition at $z = d_{\text{th}}$ should be replaced by the condition of the first kind when the temperature of the lower interface is known (as in the case of the ice-water interface). The heat flux q_{rc} is the mid-infrared cooling due to thermal radiation of the snow or ice surface in the atmospheric transparency window of $\lambda_{\text{w1}} < \lambda < \lambda_{\text{w2}}$ ($\lambda_{\text{w1}} = 8$ μm, $\lambda_{\text{w2}} = 13$ μm) [Raman et al. 2014, Chen et al. 2016, Hossain and Gu 2016]. Of course, the latent heat of ice melting, $L = 0.34$ MJ/kg, should be taken into account in the calculations. This can be done using an equivalent additional heat capacity in a narrow temperature interval near the melting temperature as it was done by Dombrovsky et al. [2019] (see [Dombrovsky and Kokhanovsky 2021] for more details). It is usually difficult to choose a realistic initial profile of temperature, $T_{\text{init}}(z)$, for the heat transfer calculations. Fortunately, the effect of this temperature profile decreases with time. As an example, it was shown by Dombrovsky et al. [2019] that the choice of this temperature profile makes no difference for the snow layer after about four hours from the initial time moment.

SOLUTION FOR ICE-COVERED LAKE

In the case of ice not covered by snow, sunlight penetrates through the ice layer, and water heating is favorable for microalgae life under ice [Write 1964, Kelley 1997, Song et al. 2019]. In the early work by Matthews and Heaney [1987], the effect of solar heating on the temperature field in an ice-covered lake was considered and a model taking into account the natural convection of water was proposed. Unfortunately, the transfer of solar radiation in water and even in a layer of ice was described using the exponential Bouguer law, which is applicable only in the case of direct incident radiation and single scattering of light in the medium. This methodological drawback remained in many recent papers [Aslamov et al. 2014, Kirillin et al. 2012, 2021, Leppäranta 2015]. This simplified model for the radiative transfer was partially compensated for by the selection of an extinction coefficient, which provided a satisfactory agreement between the calculations and the field measurements. The approach to radiative transfer used in limnology studies is insufficient, and the development of methods based on present-day models is relevant. The fact is that multiple scattering by microcracks and gas bubbles increases significantly the extinction of solar radiation in the ice layer and leads to stronger solar heating of ice. A small part of visible and short-wave infrared radiation is reflected from the ice surface, but most of this radiation in the spectral range of semitransparency is scattered and absorbed in the ice and lake water. On the contrary, the long-wave part of near-infrared solar radiation is completely absorbed in a thin surface layer of ice.

According to [Hale and Querry 1973], the absorption index of water increases with the wavelength from $\kappa_{\text{w}} = 10^{-9} - 2 \times 10^{-9}$ at $\lambda = 0.4 - 0.5$ μm to $\kappa_{\text{w}} = 10^{-5}$ at $\lambda = 1.2$ μm. The corresponding water absorption coefficient, defined as $\alpha_{\lambda}^{\text{w}} = 4\pi\kappa_{\text{w}}/\lambda$, changes from $\alpha_{\lambda}^{\text{w}} = 0.025 - 0.063$ m^{-1} to $\alpha_{\lambda}^{\text{w}} = 100$ m^{-1}. The latter means that visible radiation penetrates water to a depth of tens of meters, while infrared radiation with wavelength $\lambda_{*} = 1.2$ μm penetrates to a depth of less than 1cm. In the wavelength range $0.6 < \lambda < 1.2$ μm, the absorption index of ice differs slightly from that of water [Warren and Brandt 2008], and a similar situation occurs for ice. That is why the threshold value of λ_{*} is considered as a conventional boundary of the range of semitransparency.

The influence of uncertainty in the experimental values of the ice absorption index on the computational data for solar heating of snow and ice has been analyzed by Dombrovsky and Kokhanovsky [2022, 2023]. It was shown that the discrepancy between the data of [Warren and Brandt 2008] and [Picard et al. 2016] at $\lambda < 0.6$ μm range does not significantly affect the results of calculations. Therefore, the more complete spectral data of Warren and Brandt [2008] are used in the present work.

The gas bubbles in ice are assumed to be spherical [Alley and Fitzpatric 1999] and the radii of bubbles, a, are much greater than the wavelength. The corresponding diffraction parameter $x = 2\pi a/\lambda \gg 1$, and therefore the geometrical optics approximation can be used instead of the rigorous Mie theory [Bohren and Huffman 1998, Dombrovsky and Baillis 2010]. According to [Dombrovsky 2004], the absorption coefficient of ice which contains bubbles can be calculated as follows:

$$\alpha_\lambda = (1 - f_\mathrm{v})\alpha_\lambda^0, \tag{2}$$

where f_v is the volume fraction of bubbles and $\alpha_\lambda^0 = 4\pi\kappa_\mathrm{ice}/\lambda$ is the absorption coefficient of ice without bubbles. In most real cases, $f_\mathrm{v} \ll 1$ and $\alpha_\lambda \approx \alpha_\lambda^0$. Not only the gas bubbles but also the average distance between them is very large compared to the wavelength of the radiation. Therefore, the hypothesis of independent scattering by single bubbles is true [Mishchenko 2014, 2018].

According to [Kirillin et al. 2012], the gas bubbles are usually not uniformly distributed in the ice layer: there are more of them in the lower part of the layer. This can be taken into account in the calculations. However, following [Dombrovsky and Kokhanovsky 2020], we consider a uniform medium with only one scattering parameter $S = f_\mathrm{v}/a_{32}$ is used, where a_{32} is the Sauter's mean radius of bubbles, and the transport scattering coefficient of ice with gas bubbles is approximated as follows [Dombrovsky 2004]:

$$\sigma_\lambda^\mathrm{tr} = 0.675\ (n_\mathrm{ice}(\lambda) - 1)\ S. \tag{3}$$

This value is usually greater than the absorption coefficient in the spectral range under consideration. In other words, the extinction of sunlight in an ice layer is determined not by absorption, but by the scattering of light by gas bubbles.

Following the study by Dombrovsky and Kokhanovsky [2023], the present paper is focused primarily on modeling the thermal regime of mountain lakes like those of the Qinghai–Tibet Plateau, located at a height of about four kilometers above sea level, for which the problem is somewhat easier: it can be assumed that the sky is clear and only a small part of the light is scattered by the atmosphere. As a result, it is sufficient to consider the direct solar irradiation. Minor atmospheric precipitation and strong winds [Wang et al. 2022] result in the absence of snow cover on the ice surface. This leads to a significant solar heating of the ice and water in the lake.

The problem is simplified because in the wavelength range of $\lambda_\text{uv-vis} < \lambda < \lambda_*$ ($\lambda_\text{uv-vis} \approx 0.4$ μm) the refractive indices of water and ice differ only slightly. As a result, the reflection and refraction of light at the ice-water interface are small and can be neglected. The effect of solar light reflection at the ice-water interface is even smaller. The calculations for the ice melting season made after publication of paper by Dombrovsky and Kokhanovsky [2023] showed that the reflection of the sunlight from the illuminated ice surface can be also disregarded without any significant loss of calculation accuracy, whereas the refraction of light at an oblique illumination of ice should be taken into account.

It is assumed that optical properties of ice do not change along the horizontal ice surface and the radiative transfer along the surface of an ice-covered lake may not be considered. The 1D model for the propagation of obliquely incident radiation in an ice layer with gas bubbles has been considered recently by Dombrovsky and Kokhanovsky [2022]. It is assumed that there are no any bubbles in water, and scattering by a small amount of plankton is insignificant. In this case, the radiative transfer in the ice layer does not depend on the propagation of light in water. It is convenient to write the transport RTE and the boundary conditions at an oblique illumination of the ice layer in dimensionless variables [Dombrovsky and Kokhanovsky 2023]:

$$\mu \frac{\partial \bar{I}_\lambda}{\partial \tau_\lambda^\mathrm{tr}} + \bar{I}_\lambda = \frac{\omega_\lambda^\mathrm{tr}}{2}\bar{G}_\lambda, \quad \bar{G}_\lambda(\tau_\lambda^\mathrm{tr}) = \int_{-1}^{1} \bar{I}_\lambda(\tau_\lambda^\mathrm{tr}, \mu)\,\mathrm{d}\mu \tag{4a}$$

$$\bar{I}_\lambda(0, \mu) = \delta(\mu_\mathrm{j} - \mu), \quad \bar{I}_\lambda(\tau_{\lambda,0}^\mathrm{tr}, -\mu) = 0, \quad \mu, \mu_\mathrm{j} > 0, \tag{4b}$$

where $\bar{I}_\lambda = I_\lambda / I_\lambda^{\mathrm{inc}}$, $\bar{G}_\lambda = G_\lambda / I_\lambda^{\mathrm{inc}}$, I_λ^{inc} is the spectral intensity of incident radiation in direction $\mu_\mathrm{i} = \cos\theta_\mathrm{i}$ (θ_i is measured from the external normal), $\mu_\mathrm{j} = \sqrt{1 - (1 - \mu_\mathrm{i}^2)/n_{\mathrm{ice}}^2}$ is the cosine of the refraction angle, $\tau_\lambda^{\mathrm{tr}}(z) = \int_0^z \beta_\lambda^{\mathrm{tr}}(z)\,\mathrm{d}z$ is the optical depth and $\tau_{\lambda,0}^{\mathrm{tr}} = \tau_\lambda^{\mathrm{tr}}(d)$. The intensity of radiation and the spectral irradiation are presented as follows:

$$\bar{I}_\lambda = \bar{J}_\lambda + E_\lambda^{\mathrm{j}} \delta(\mu_\mathrm{j} - \mu), \quad E_\lambda^{\mathrm{j}} = \exp\left(-\tau_\lambda^{\mathrm{tr}}/\mu_\mathrm{j}\right), \quad \bar{G}_\lambda = \bar{G}_\lambda^{\mathrm{dif}} + E_\lambda^{\mathrm{j}}, \quad \bar{G}_\lambda^{\mathrm{dif}} = \int_{-1}^{1} \bar{J}_\lambda \,\mathrm{d}\mu \tag{5}$$

The resulting problem for the diffuse component of the radiation intensity is:

$$\mu \frac{\partial \bar{J}_\lambda}{\partial \tau_\lambda^{\mathrm{tr}}} + \bar{J}_\lambda = \frac{\omega_\lambda^{\mathrm{tr}}}{2} \bar{G}_\lambda, \quad \bar{J}_\lambda(0, \mu) = \bar{J}_\lambda(d, -\mu) = 0, \quad \mu > 0\,. \tag{6}$$

The two-flux approximation gives the following boundary-value problem for the diffuse irradiation:

$$-\left(\bar{G}_\lambda^{\mathrm{dif}}\right)'' + \xi_\lambda^2 \bar{G}_\lambda^{\mathrm{dif}} = 4\omega_\lambda^{\mathrm{tr}} E_\lambda^{\mathrm{j}}, \quad \xi_\lambda = \sqrt{1 - \omega_\lambda^{\mathrm{tr}}} \tag{7a}$$

$$\tau_\lambda^{\mathrm{tr}} = 0, \quad \left(\bar{G}_\lambda^{\mathrm{dif}}\right)' = 2\bar{G}_\lambda^{\mathrm{dif}}, \qquad \tau_\lambda^{\mathrm{tr}} = \tau_{\lambda,0}^{\mathrm{tr}}, \quad \left(\bar{G}_\lambda^{\mathrm{dif}}\right)' = -2\bar{G}_\lambda^{\mathrm{dif}}. \tag{7b}$$

This problem statement is correct for any variation of the optical properties across the ice layer. In the case of uniform optical properties, one can obtain the analytical solution at $\xi_\lambda \neq \nu_\mathrm{j} = 1/\mu_\mathrm{j}$:

$$\bar{G}_\lambda^{\mathrm{dif}} = \frac{4\omega_\lambda^{\mathrm{tr}}}{\xi_\lambda^2 - \nu_\mathrm{j}^2} \left(E_\lambda^{\mathrm{j}} + \frac{2 - \nu_\mathrm{j}}{2 - \xi_\lambda} \frac{A E_\lambda^{\mathrm{dif}} + B / E_\lambda^{\mathrm{dif}}}{\left(E_{\lambda,0}^{\mathrm{dif}}\right)^2 - \zeta_\lambda^2} \right), \quad \zeta_\lambda = \frac{2 + \xi_\lambda}{2 - \xi_\lambda} \tag{8a}$$

$$A = \psi \zeta_\lambda - E_{\lambda,0}^{\mathrm{j}} E_{\lambda,0}^{\mathrm{dif}}, \quad B = E_{\lambda,0}^{\mathrm{dif}} \left(\zeta_\lambda E_{\lambda,0}^{\mathrm{j}} - \psi E_{\lambda,0}^{\mathrm{dif}} \right), \quad \psi = \frac{2 + \nu_\mathrm{j}}{2 - \nu_\mathrm{j}} \tag{8b}$$

$$E_{\lambda,0}^{\mathrm{j}} = \exp\left(-\nu_\mathrm{j} \tau_{\lambda,0}^{\mathrm{tr}}\right), \quad E_\lambda^{\mathrm{dif}} = \exp(-\xi_\lambda \tau_\lambda^{\mathrm{tr}}), \quad E_{\lambda,0}^{\mathrm{dif}} = \exp\left(-\xi_\lambda \tau_{\lambda,0}^{\mathrm{tr}}\right). \tag{8c}$$

The obvious solution to the radiative transfer problem in a layer of water is:

$$\bar{I}_\lambda^{\mathrm{w}}(z, \mu) = \begin{cases} \bar{I}_\lambda(d, \mu) \exp(-\tau_\lambda^{\mathrm{w}}/\mu) & \text{when } \mu > 0 \\ 0 & \text{when } \mu < 0 \end{cases}, \quad \tau_\lambda^{\mathrm{w}} = \alpha_\lambda^{\mathrm{w}} \times (z - d) \tag{9}$$

According to Eq. (9), the intensity of light in water at $\mu = 1$ decreases most slowly. As a result, with increasing depth the light becomes less diffuse and closer to directed vertically downward. The solar radiation power absorbed in ice and water can be calculated as follows:

$$P(z) = \int_{\lambda_{\text{uv-vis}}}^{\lambda_*} p(\lambda, z)\,\mathrm{d}\lambda, \quad p(\lambda, z) = \begin{cases} \alpha_\lambda G_\lambda(z) & \text{when } z \le d \\ \alpha_\lambda^{\mathrm{w}} G_\lambda^{\mathrm{w}}(z) & \text{when } z > d \end{cases} \tag{10}$$

The functions in the right-hand side of the second of these equations are defined as:

$$G_\lambda(z) = I_{\lambda,\mathrm{i}}^{\mathrm{inc}} \times \left\{ \bar{G}_\lambda^{\mathrm{dif}}(z) + \exp\left(-\nu_\mathrm{j} \beta_\lambda^{\mathrm{tr}} z\right) \right\}, \tag{11a}$$

$$G_\lambda^{\mathrm{w}}(z) = I_{\lambda,\mathrm{i}}^{\mathrm{inc}} \times \left\{ \bar{G}_\lambda^{\mathrm{dif}}(d) \exp(-2\tau_\lambda^{\mathrm{w}}) + \exp\left[-\nu_\mathrm{j} (\beta_\lambda^{\mathrm{tr}} d + \tau_\lambda^{\mathrm{w}})\right] \right\}. \tag{11b}$$

The diffuse irradiation component in water decreases with depth twice as fast as the directional one. It is clear that Bouguer's law with a constant extinction coefficient in the exponent, which was applied for the light propagation in water under an ice layer in limnology studies, is not correct.

PROFILES OF ABSORBED RADIATION POWER IN ICE AND WATER

The above solution was used to calculate profiles of absorbed radiation power in ice and water of the Ngoring Lake at the end of March (the time of the beginning of ice melting). The spectral radiative flux at the surface of Ngoring Lake (34.5–35.5°N and 97–98°E, the lake's altitude is ≈4,300 m a.s.l.) has been calculated in [Dombrovsky and Kokhanovsky 2023]. The calculated profiles of the radiation power absorbed in the ice layer and in the water under the ice at the zenith angle of the Sun 60$^\mathrm{o}$ are shown in Fig. 1. First of all, a significant attenuation of light in the ice layer draws attention. Increasing the scattering parameter leads to an increase in the absorption of solar radiation in the upper part of the ice layer and a significant decrease in the absorption in water under the ice. When ice melts, accompanied by a decrease in the thickness of the ice cover, the absorption of radiation in the ice layer decreases considerably, whereas the absorption of radiation in water increases very strongly. The latter is the physical cause of an observation by Lazhu et al. [2021] for several lakes in the Tibetan Plateau where the water temperature at some distance from the ice-water interface increased rapidly during ice melting.

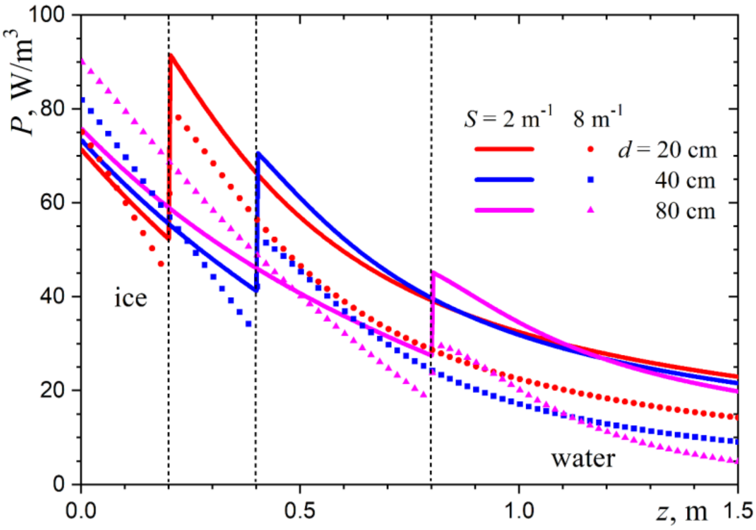

Figure 1. Absorbed radiation power profiles in ice and water at different values of scattering parameter and ice thickness.

Before calculating the heating and melting of ice on the lake surface, let us consider the peculiarities of the temperature stratification of water in the lake. The temperature field in the lake water is largely determined by the non-monotonic temperature dependence of the water density with a maximum density $\rho_\mathrm{w}^* = 1000$ kg/m^3 at temperature $T_* = 4$ °C [Matthews and Heaney 1987]:

$$\rho_\mathrm{w}(T) = \rho_\mathrm{w}^* \times \{1 - \varphi \times (T - T_*)^2\}, \quad \varphi = 8 \times 10^{-6} \ \ {}^\circ\mathrm{C}^{-2}. \tag{12}$$

At the beginning of ice formation, the water temperature at the ice-water interface becomes equal to $T_0 = 0$ °C, while the water at depth may remain warmer. However, even at the bottom of a deep lake, the water temperature cannot be higher than T_*. In a severe winter, the lake can freeze to the bottom, but this paper considers early spring, when the ice thickness does not exceed 1 m.

Field observations of Ngoring Lake and some other lakes in the Qinghai–Tibet Plateau have shown an interesting thermal regime, named "anomalous winter" by Kirillin et al. [2021], when at a depth of 1.5–3 m under the ice layer the water is heated by solar radiation to a temperature $T_\mathrm{max} > T_*$. Solar radiation absorbed in the upper water layer leads to an increase in T_max, and this effect was measured by Kirillin et al. [2021] in March and early April. An estimate of the heat flux from relatively warm water to the ice layer in [Dombrovsky and Kokhanovsky, 2023] gave the value $q_\text{w-ice} = 1.1 - 1.2$ kW/m^2, which agrees with measurements for Baikal Lake by Aslamov et al. [2014]. However, the effect of this heat flux turned out to be much smaller than ice heating by solar radiation.

HEAT TRANSFER AND BEGINNING OF ICE MELTING

The heat transfer on the illuminated ice surface changes significantly during the day but this does not affect the temperature of ice at some distance from the surface because of the large heat capacity of the ice layer. An estimate based on the Fourier criterion confirm that a thermal relaxation time for the 0.5 m thick ice is about ten days. This allows using a steady-state model with constant heat transfer parameters, varying according to weather changes from week to week.

The ice melting at Ngoring Lake in March is very slow, and only in April does ice melting accelerate, completed by April 16 [Kirillin et al. 2021]. This result is clear from the observations by Zhou et al. [2022]: the air temperature in March is almost constant and does not exceed -10°C, while it increases up to -3°C in the middle of April at the lake. Therefore, the analysis of ice melting on the surface of Ngoring Lake should refer to the conditions of the first half of April, when the day-averaged solar illumination of the lake practically does not change and the air temperature is the only changing parameter of the problem. According to [Wang et al. 2022], the wind speed in the first two weeks of April is equal to 4 m/s and not changed during the day. The corresponding value of the heat transfer coefficient is about $h = 20$ W/(m^{2}K) [Defraeye et al. 2011, Mirsadeghi et al. 2014].

The boundary-value problem for the quasi-steady temperature profile in the ice layer is as follows:

$$k_{\mathrm{ice}}T'' + \bar{P}(z) = 0, \quad 0 < z < d \tag{13a}$$

$$k_{\mathrm{ice}}T'(0) = \bar{q}_{\mathrm{inf}}^{\mathrm{sol}} - q_{\mathrm{conv}} - q_{\mathrm{rc}}, \quad T(d) = T_0, \tag{13b}$$

where

$$\bar{P}(z) = \frac{1}{t_{\mathrm{day}}}\int_0^{t_{\mathrm{day}}} P(z,t)\,\mathrm{d}t, \quad \bar{q}_{\mathrm{inf}}^{\mathrm{sol}} = \frac{1}{t_{\mathrm{day}}}\int_0^{t_{\mathrm{day}}} q_{\mathrm{inf}}^{\mathrm{sol}}(t)\,\mathrm{d}t, \quad q_{\mathrm{conv}} = h(T(0) - T_{\mathrm{air}}) \tag{14a}$$

$$q_{\mathrm{inf}}^{\mathrm{sol}}(t) = \int_{\lambda_*}^{\infty} q_{\mathrm{i},\lambda}^{\mathrm{inc}}(\lambda,t)\,\mathrm{d}\lambda, \quad q_{\mathrm{rc}} = \pi\int_{\lambda_{\mathrm{w1}}}^{\lambda_{\mathrm{w2}}} I_{\lambda,\mathrm{b}}\big(T(0)\big)\,\mathrm{d}\lambda. \tag{14b}$$

Here $q_{\mathrm{i},\lambda}^{\mathrm{inc}}$ is the incident radiative flux at the solar zenith angle θ_{i}. To determine the function $\bar{P}(z) = P(z) \times t_{\mathrm{dl}}/t_{\mathrm{day}} = P(z)/2$ one can use the average profiles of $P(z)$ for daylight hours. The same relation is true for the solar infrared radiation flux, taking into account its contribution to the integral radiative flux. The value $\bar{q}_{\mathrm{inf}}^{\mathrm{sol}} = 37$ W/m^2 is used in the calculations. It is also convenient to use the following approximation for the temperature dependence of q_{rc}:

$$q_{\mathrm{rc}} = q_0 + \eta \times (T(0) - T_0), \quad q_0 = 93 \ \ \mathrm{W/m^2}, \quad \eta = 1.6\ \mathrm{W/(m^2\,K)}. \tag{15}$$

The radiative cooling of ice compensates the radiative flux to the ice surface in the opacity range, and the infrared solar heating cannot lead to the surface ice melting.

The condition for the beginning of ice melting can be obtained using the analytical solution to the problem (13a,b):

$$T(z) = T_0 + \frac{f_2(d) - f_2(z)}{k_{\mathrm{ice}}} - \frac{Q}{k_{\mathrm{ice}}}(d - z) \tag{16a}$$

$$f_1(z) = \frac{1}{2}\int_0^z P(z)\mathrm{d}z, \quad f_2(z) = \int_0^z f_1(z)\mathrm{d}z, \quad Q = q_{\mathrm{conv}} + q_{\mathrm{inf}}^{\mathrm{ice}} - \bar{q}_{\mathrm{inf}}^{\mathrm{sol}}. \tag{16b}$$

The ice melting on its lower surface begins when $T'(d) = 0$. This enables us to obtain the threshold temperature profile in the ice layer:

$$T(z) = T_0 + \frac{f_2(d) - f_2(z)}{k_{\mathrm{ice}}} - \frac{f_1(d)}{k_{\mathrm{ice}}}(d - z). \tag{17}$$

The calculated temperature profiles are shown in Fig. 2. As one might expect, scattering plays a significant role in the case of a thick ice layer and the assumption of uniform distribution of gas bubbles in the ice is acceptable only for ice layers less than 0.5 m thick.

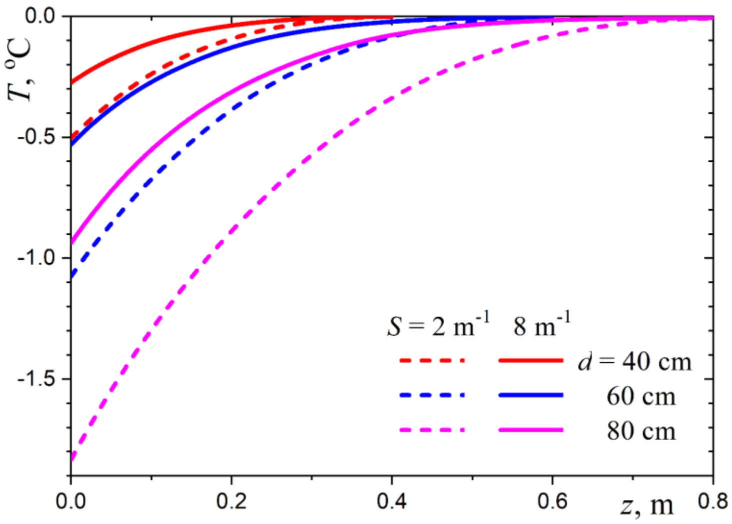

Figure 2. Temperature profiles in ice layers of different thicknesses at the onset of melting on the ice-water interface.

One can also calculate the temperature of the illuminated ice surface, at which melting begins at the ice-water interface:

$$T^*_{\mathrm{surf}} = T_0 + \frac{f_2(d) - f_1(d)d}{k_{\mathrm{ice}}}. \tag{18}$$

Typical dependences of $T^*_{\mathrm{surf}}(d)$ are plotted in Fig. 3. As one might expect, the effect of light scattering by gas bubbles is more significant for thick ice layers.

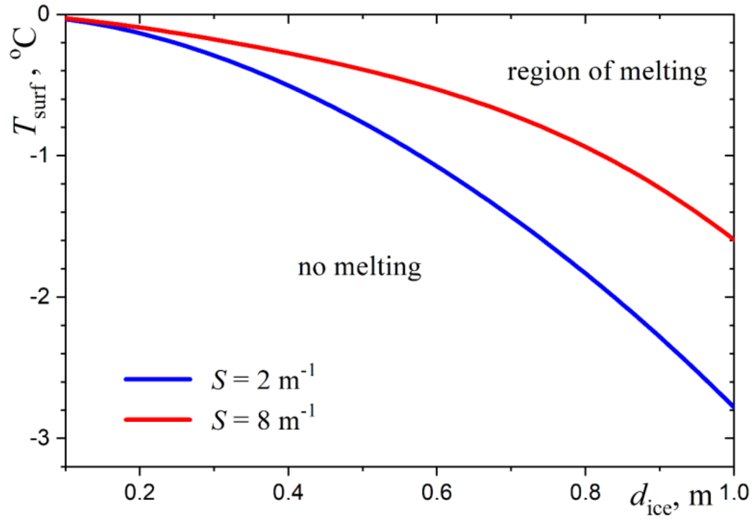

Figure 3. Dependences of the surface temperature threshold value on the ice thickness.

EFFECT OF A SNOW LAYER ON LAKE ICE MELTING

In the absence of snow cover on the lake ice surface, solar radiation penetrates through the semitransparent ice and already in the beginning of spring significantly heats up the water under the ice. Interestingly, the lake ice, even with a thickness of about one meter, begins to melt from the lower surface. Calculations have shown that this is not due to heating of the ice by the warmer water, but almost exclusively due to solar heating of the ice. This is so because the upper layer of ice is continuously cooled by the colder air, as well as by radiative cooling. This cooling is not compensated by the daytime heating of the ice surface by the infrared radiation of the Sun but does not prevent heating of the lower part of the ice layer and the beginning of spring ice melting.

The discussed thermal regime of ice on the lake surface and the ice melting under the action of spring solar heating changes radically in the presence of snow on the ice surface. The effect of snow is due to two main factors: firstly, snow significantly reduces the solar radiative flux on the ice surface due to strong scattering of radiation and, secondly, the thermal conductivity of snow is so small that even a thin layer of snow protects the ice surface from convective and radiative cooling. The physical model of snow's effect on ice melting should include not only the transfer of solar radiation in the snow and ice layer but also the heat conduction process. If we do not take into account changes in the structure and optical properties of snow during its heating, the above physical problems can be solved sequentially: first (but with repetition of the calculation as the Sun moves across the sky) to calculate radiative transfer, and then to solve the heat transfer problem taking into account the absorption of solar radiation both on the snow surface and in the volume of snow and ice. Of course, in the thermal part of the computational model, it is necessary to consider the convective heat transfer with the surrounding air, as well as the mid-infrared radiative cooling of the snow surface.

Let us first consider the propagation of direct solar radiation through the snow layer. In order not to complicate the solution, we will not take into account the scattered radiation from a cloudless sky. Nor will we take into account that there is ice under the snow, which also scatters sunlight. In fact, ice with air bubbles scatters light much more weakly than snow, and only a small fraction of the light transmitted through the snow is scattered by the ice in the direction of the snow. In the case of a thin snow layer, the scattering of radiation in the ice layer has some influence on the radiative transfer in the snow layer, but this does not affect the main results of the calculations presented below.

When calculating radiative transfer in a medium with multiple scattering, the transport approximation is used. In addition, the radiation intensity is represented in the form of two additive components: direct radiation and a diffuse component formed due to the scattering of radiation in the medium. The use of the two-flux method leads to the boundary-value problem for the normalized diffuse irradiation. The analytical solution for a uniform snow layer looks similar to (8a-c), but with replacement of ν_j by ν_i:

$$\bar{G}_\lambda^\mathrm{dif} = \frac{4\omega_\lambda^\mathrm{tr}}{\xi_\lambda^2 - \nu_\mathrm{i}^2}\left(E_\lambda^\mathrm{i} + \frac{2-\nu_\mathrm{i}}{2-\xi}\frac{AE_\lambda^\mathrm{dif} + B/E_\lambda^\mathrm{dif}}{\left(E_{\lambda,0}^\mathrm{dif}\right)^2 - \zeta_\lambda^2}\right), \quad \zeta_\lambda = \frac{2+\xi_\lambda}{2-\xi_\lambda} \tag{19a}$$

$$A = \psi\zeta_\lambda - E_{\lambda,0}^\mathrm{i} E_{\lambda,0}^\mathrm{dif}, \quad B = E_{\lambda,0}^\mathrm{dif}\left(\zeta_\lambda E_{\lambda,0}^\mathrm{i} - \psi E_{\lambda,0}^\mathrm{dif}\right), \quad \psi = \frac{2+\nu_\mathrm{i}}{2-\nu_\mathrm{i}} \tag{19b}$$

$$E_{\lambda,0}^\mathrm{i} = \exp\left(-\nu_\mathrm{i}\tau_{\lambda,0}^\mathrm{tr}\right), \quad E_\lambda^\mathrm{dif} = \exp\left(-\xi_\lambda\tau_\lambda^\mathrm{tr}\right), \quad E_{\lambda,0}^\mathrm{dif} = \exp\left(-\xi_\lambda\tau_{\lambda,0}^\mathrm{tr}\right). \tag{19c}$$

In the limit of $\tau_{\lambda,0}^\mathrm{tr} \to \infty$, this solution coincides with that derived by Dombrovsky et al. [2019].

Note that the so-called plane albedo is widely used in studies on snow optics [Kokhanovsky, 2021]. This is a spectral quantity coinciding with directional hemispherical reflectance and it is denoted as $R_\lambda^\mathrm{d-h}$ in works on the optical properties of scattering materials [Baillis and Sacadura 2000, Sacadura 2011]. According to [Dombrovsky and Baillis, 2010], the plane albedo can be determined as:

$$R_\lambda^\mathrm{d-h} = \nu_\mathrm{i}\,\bar{G}_\lambda^\mathrm{dif}(0)/2. \tag{20}$$

More details on the definitions and terminology used for the reflection characteristics can be found in [Schaepman-Staub et al. 2006]. When illumination is normal to the snow surface, the value $R_\lambda^\mathrm{d-h}$ is denoted by $R_\lambda^\mathrm{n-h}$ and is called the normal snow albedo. The analytical solution obtained for $\bar{G}_\lambda^\mathrm{dif}$ gives the following formula for the normal snow albedo in the limit of $\tau_{\lambda,0}^\mathrm{tr} \to \infty$:

$$R_\lambda^\text{n-h} = \frac{2\omega_\lambda^\mathrm{tr}}{(1+\xi)(2+\xi)} = \frac{\omega_\lambda^\mathrm{tr}}{\left(1+\sqrt{1-\omega_\lambda^\mathrm{tr}}\right)\left(1+2\sqrt{1-\omega_\lambda^\mathrm{tr}}\right)}. \tag{21}$$

This formula is a particular case of a general relation for the refractive medium derived by Dombrovsky et al. [2006] and applied in [Dombrovsky and Kokhanovsky [2020] to calculate the spectral albedo of ice.

The optical properties of snow were calculated in the same way as in [Dombrovsky et al. 2019] (see also the overview by Dombrovsky and Kokhanovsky [2021]). As in [Dombrovsky et al. 2019], it was assumed that the ice grains have a mean radius of $a = 100$ μm and their volume fraction is equal to $f_\mathrm{v} = 0.33$. Not that the ice grains in snow satisfy the conditions of the independent scattering.

The effect of snow layer thickness on the normal albedo $R_\lambda^\text{n-h}$ and the directional-hemispherical transmittance $T_\lambda^\mathrm{d-h}$ of snow is shown in Fig. 4. The calculations of the transmittance were made for solar zenith angle $\theta_\mathrm{i} = 60^\circ$. Increasing the snow thickness leads to an increase in albedo, but the limiting value of the normal albedo is reached only at $d_\mathrm{snow} \sim 50$ mm. It is also important that the maximum albedo of snow strongly depends on the wavelength (see book by Kokhanovsky [2021] for details). At the same time, even a 5 mm thick snow layer transmits only about 10–14 % of the incident radiation and the value of $T_\lambda^\mathrm{d-h}$ is small at every wavelength in the transparency range. The latter result means that a snow layer more than about 5 mm thick practically does not transmit solar radiation to the ice surface.

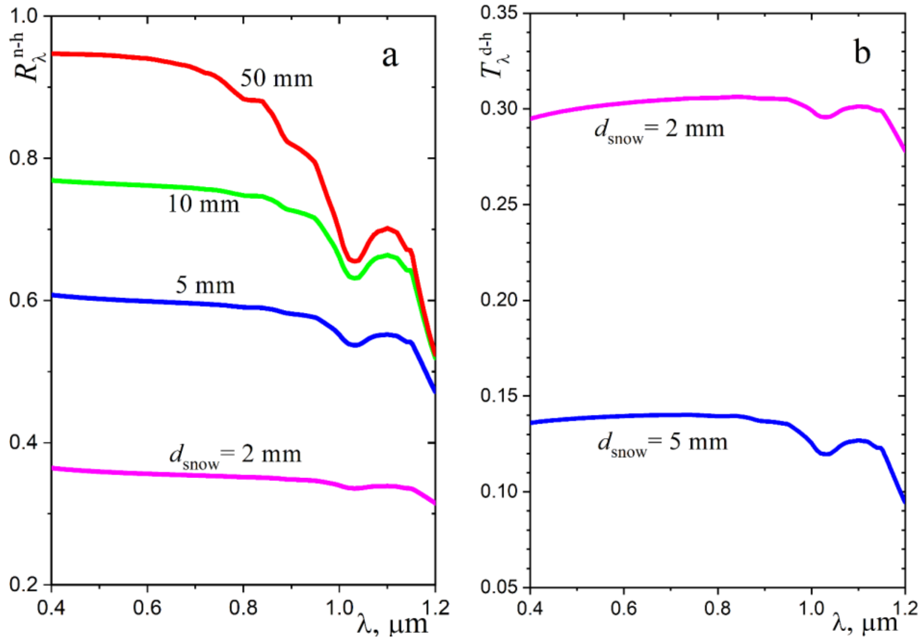

Figure 4. Effect of snow layer thickness on (a) normal albedo and (b) transmittance of snow.

Consider now the heat transfer problem for solar heating and possible snow melting on the ice surface, taking into account the conductive heating of the ice layer. The temperature of the lower surface of the ice layer (at the ice-water interface) is 0°C. This boundary condition allows one not to consider the heat transfer under the ice, which is necessary when the ice layer is located, for example, on bare ground. The calculations of solar radiation propagation in the snow layer showed that even a snow layer with a thickness of less than one centimeter is opaque to sunlight and the absorption of light passing through the snow to the ice layer can be neglected. Note that this result agrees well with the measurements by Perovich [2007]. In addition, a small reflection of light from the ice surface and a small scattering of light by gas bubbles in the ice allows a significant simplification of the radiative transfer problem: one can assume that the sunlight transmitted through the snow layer does not return. At the same time, when analyzing the thermal state of the snow layer illuminated by the Sun, it is necessary to take into account the heat conducted away from the snow into the ice layer. In other words, the energy equation should be solved in the computational domain including both the snow cover and the ice layer:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + P, \quad t > 0, \quad 0 < z < d_{\mathrm{th}} = d_{\mathrm{snow}} + d_{\mathrm{ice}} \tag{22a}$$

$$T(0, z) = T_{\mathrm{init}}(z) \tag{22b}$$

$$z = 0, \quad -k\frac{\partial T}{\partial z} = h\,(T_{\mathrm{air}} - T) + q_{\mathrm{inf}}^{\mathrm{sol}} - \varepsilon\, q_{\mathrm{rc}}, \qquad z = d_{\mathrm{th}}, \quad T(t, z) = T_0 \tag{22c}$$

$$q_{\mathrm{inf}}^{\mathrm{sol}} = \int_{\lambda_*}^{\lambda_{\mathrm{op}}} q_\lambda^{\mathrm{sol}}(t)\, \mathrm{d}\lambda, \qquad q_{\mathrm{rc}} = \pi \int_{\lambda_{\mathrm{w1}}}^{\lambda_{\mathrm{w2}}} I_{\lambda,\mathrm{b}}(T, \lambda)\, \mathrm{d}\lambda. \tag{22d}$$

where $T_0 = 0$°C, $q_{\mathrm{inf}}^{\mathrm{sol}}(t)$ is the integral infrared solar radiative flux in the opacity range of $\lambda_* < \lambda < \lambda_{\mathrm{op}}$ (the upper boundary of this range was taken equal to $\lambda_{\mathrm{op}} = 2.4$ μm). Specific volumetric heat capacity ρc and thermal conductivity k of the medium are:

$$\rho c = \begin{cases} (\rho c)_{\mathrm{snow}} & \text{when } 0 < z < d_{\mathrm{snow}} \\ (\rho c)_{\mathrm{ice}} & \text{when } d_{\mathrm{snow}} < z < d_{\mathrm{th}} \end{cases}, \qquad k = \begin{cases} k_{\mathrm{snow}} & \text{when } 0 < z < d_{\mathrm{snow}} \\ k_{\mathrm{ice}} & \text{when } d_{\mathrm{snow}} < z < d_{\mathrm{th}} \end{cases}. \tag{23}$$

The initial temperature profile was assumed to be as follows:

$$T_{\mathrm{init}}(z) = \begin{cases} T_{\mathrm{air}} & \text{when } z \le d_{\mathrm{snow}} \\ T_{\mathrm{air}} + (T_0 - T_{\mathrm{air}})\,(z - d_{\mathrm{snow}})/d_{\mathrm{ice}} & \text{when } z > d_{\mathrm{snow}} \end{cases}. \tag{24}$$

When modeling the effect of the snow layer on the opening of the lake from ice, it would be incorrect to consider the solar heating conditions typical of the high-mountain lakes of northeastern Tibet, when, due to low precipitation and constantly strong winds, the ice on these lakes is not covered by snow. In addition, at low altitudes, unlike in high mountains, not only direct solar radiation but also diffuse radiation from the light-scattering atmosphere should be taken into account. Following [Dombrovsky et al. 2019], we use the same data on solar irradiation for the summer solstice during the Arctic polar summer at latitude 70°. Taking into account the diffuse atmospheric radiation requires additional relations, which are obtained by solving the following problem:

$$(\bar{G}_\lambda)'' - \xi^2 \bar{G}_\lambda = 0 \tag{25a}$$

$$\tau_\lambda^{\mathrm{tr}} = 0, \quad (\bar{G}_\lambda)' = 2(\bar{G}_\lambda - 2), \qquad \tau_\lambda^{\mathrm{tr}} = \tau_{\lambda,0}^{\mathrm{tr}}, \quad (\bar{G}_\lambda)' = -2\bar{G}_\lambda. \tag{25b}$$

The analytical solution to this problem is given by:

$$\bar{G}_\lambda = \frac{4}{2-\xi} \frac{\zeta E_\lambda^{\mathrm{dif}} - \left(E_{\lambda,0}^{\mathrm{dif}}\right)^2 / E_\lambda^{\mathrm{dif}}}{\zeta^2 - \left(E_{\lambda,0}^{\mathrm{dif}}\right)^2}. \tag{26}$$

In the limit of $\tau_{\lambda,0}^{\mathrm{tr}} \to \infty$, equation (26) reduces to the following one: $\bar{G}_\lambda = 4\, E_\lambda^{\mathrm{dif}}/(2+\xi)$.

To exclude the influence of the initial temperature profile, we consider the calculated temperature profiles on day 14 from $t = 0$. For certainty, the ice layer thickness is assumed to be equal to $d_{\mathrm{ice}} = 40$ cm, whereas the thickness of the snow layer varies widely. The computational results presented in Fig. 5 for three values of d_{snow} differ most markedly at midnight when the snow has time to cool by 0.3–0.4 °C as a result of convective and radiative cooling. It is interesting that the maximum heating of snow occurs in the second half of the day and even at $d_{\mathrm{snow}} = 200$ mm only the surface layer of snow 2–3 mm thick melts. This result is qualitatively different from the case of thick snow cover considered in [Dombrovsky et al. 2019, Dombrovsky and Kokhanovsky 2022] due to heat sinking into the ice layer. Of course, this heat sink is more noticeable in case of a thin snow layer, but remains significant even at $d_{\mathrm{snow}} = 200$ mm. Calculations have shown that snow less than 15 mm thick does not melt at all due to cooling by the ice layer. It is important that in all the variants, the ice layer is far from complete melting.

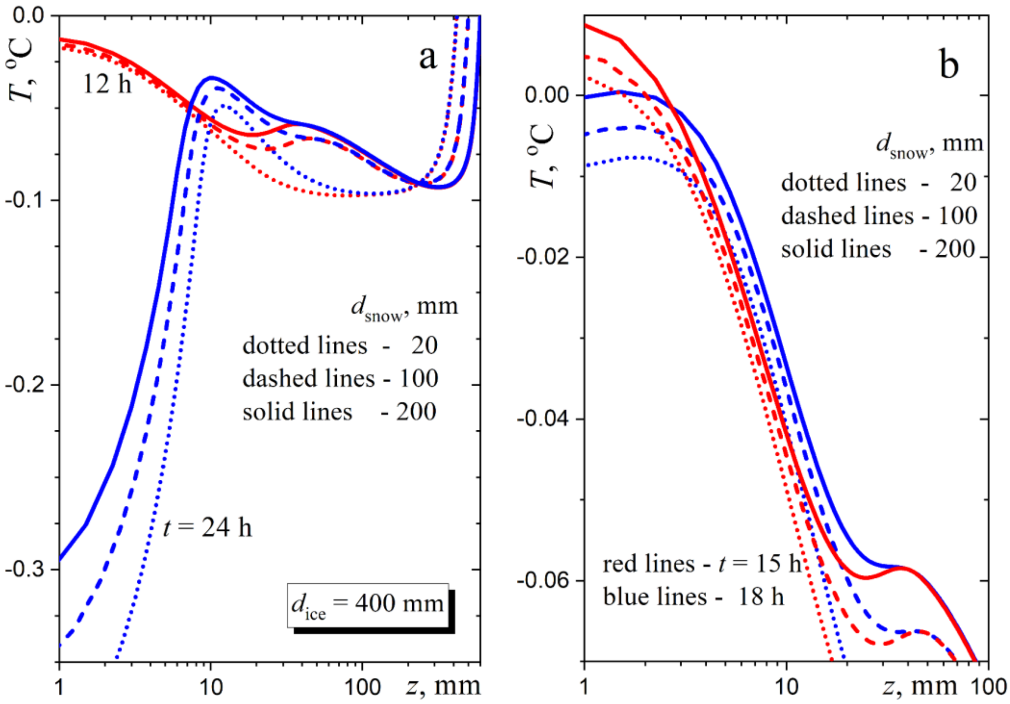

Figure 5. Temperature profiles in the snow and ice layers underneath at different times of the day: a – at noon and midnight, b – in the afternoon.

The above computational study gives the following general picture of solar heating of the ice layer covered by snow. When the upper surface of ice on a lake is illuminated by the spring Sun, the thick ice layer begins to melt even in very cold air and melting occurs from the lower surface of the ice layer. But if there is a thin layer of snow on the surface of the ice (even less than one centimeter thick) the snow does not allow much of the sunlight to penetrate into ice and makes melting of the ice surface underneath impossible. At the same time, the snow itself on the ice surface does not heat up because it scatters a significant part of the visible solar radiation, and the absorbed solar heat is almost immediately transferred to the relatively thick layer of ice. Snow melting can only begin when the snow layer is thicker than about 15 mm and only when the snow layer is 200 mm thick does melting become significant. Of course, the produced water flows down through the pores in the snow and freezes again at some distance from the ice surface. If the initial thickness of the snow layer is more than 300 mm, melting of the snow near the sunlit surface continues and leads to the formation of a kind of mushy zone on the ice with ice particles suspended in water, which tend to float to the water surface. This stage of the process can be quite long, but as a result the ice surface turns out to be covered with a layer of water.

AN ESTIMATE OF THE EFFECT OF MELT POND ON MELTING OF ICE

Meltwater, formed initially by snowmelt, does not cover the ice surface uniformly. This is evidenced by numerous observations both for ice-covered lakes and for the much better-studied Arctic Sea ice. Note that the abnormally strong (certainly partial) melting of sea ice in the Arctic during the polar summer is an extremely important process that has been intensively analyzed over the last decade. One should name several studies that are devoted to the formation and evolution of melt ponds on the Arctic Sea ice. In chronological order, these are experimental and analytical works by Polashenski et al. [2012], Hudson et al. [2013], Schröder et al. [2014], Popović et al. [2018], Malinka et al. [2018], Ma et al. [2019], König and Oppelt [2020], Perovich et al. [2021], Sterlin et al. [2021], Lei et al. [2022], and Rosenburg et al. [2023]. Photographs in the literature show numerous melt ponds and the change in the ice surface area occupied by these ponds during the polar summer. Note that similar melt ponds are also observed on glaciers [Rockström and Gaffney 2021]. In most of the above-mentioned works on sea ice, the authors are focused on positive feedback that leads to an enormous increase in the total area of the melt ponds. The point is that the albedo of snow in the visible range of the spectrum is very high, and when snow melts in polar summer, forming melt ponds, the reflection of solar radiation is radically reduced. As a result, sea ice receives much more solar heat, causing it to melt and increasing the area occupied by melt ponds. This positive feedback over a large surface area of the Arctic Sea is accompanied by an increase in the water vapor content of the atmosphere and is important for the continuation of global warming.

Even visual observations allow us to distinguish between two types of melt ponds: the so-called bright ponds and dark ponds. The ice on the bottom of the bright pond is mostly smooth and dense but with small cracks and highly light-scattering porous areas with fine pores. The bottom of the dark pond is more heterogeneous and has relatively large cracks and voids [König and Oppelt 2020]. It turns out that this difference is related to the different structures of the first-year ice and multiyear ice [Li et al. 2020]. The experimental work by König and Oppelt [2020] confirmed the qualitative results of the analytical study of melt pond reflectivity by Malinka et al. [2018]. Note that the work by Malinka et al. [2018] was based on the same methodological framework as the present paper, including the transport approximation and the two-flux model for radiative transfer.

For simplicity, we will focus on a more simple melting problem for the first-year ice. The formation of a layer of water on the ice surface and the subsequent melting of ice are complex processes that deserve special modeling. Nevertheless, it is possible to suggest a simple physical model for ice melting under a meltwater layer. It is obvious that on both surfaces of the ice layer, the temperature is constant and equal to 0^{o}C. Of course, the total radiation power absorbed in the ice volume leads not to increase the ice temperature, but to its melting. The heat flux from water under the ice, as was shown above, is insignificant. Indeed, there is a stable temperature stratification of water directly under the ice, and the thermal conductivity of water is rather small. On the contrary, the natural convection of water in a melt pond can give a noticeably larger contribution to ice melting, and this should be taken into consideration. This effect is more significant in the case of a small ice layer thickness.

The temperature profile along the depth of the melt pond is non-monotonic and has a maximum at some depth below the surface. Indeed, the heating of the water surface by infrared solar radiation is compensated by convective cooling and radiative cooling in the middle-infrared transparency window of the cloudless atmosphere. Even at temporary heating of the surface layer of water to the temperature

$T_* = 4$°C, this water, as denser, goes down to some depth $d_* < d_\mathrm{p}$, where d_p is the depth of the melt pond. At the bottom of the melt pond, at a depth of $d_* < d < d_\mathrm{p}$, the water temperature decreases from T_* at $d = d_*$ to $T_0 = 0$°C at the ice surface. Intensive natural convection takes place in this lower layer. Obviously, most of the solar radiative flux absorbed in the water volume is transferred to the ice in this lower layer of the melt pond. This is so because in the upper layer of the pond water (at a depth of $d < d_*$) due to stable temperature stratification, the only heat transfer mode – heat conduction – operates, and the thermal conductivity of water is small: $k_\mathrm{w} = 0.57\ \mathrm{W/(m\ K)}$. As a result, for example, at $d - d_* = 10$ cm and temperature difference $T_* - T_0 = 4$°C the upward heat flux is very small: $q_\mathrm{cond} \approx 23$ W/(m^2 K). Therefore, in the suggested approximate model, it is assumed that the main part of solar radiation in the spectral range of water semitransparency absorbed in the volume of the melt pond is transferred to the ice layer. We will consider only the day-averaged radiative flux at the melt pond surface. The change of heat transfer conditions on the surface of the melt pond during the day does not matter for the considered ice melting because of the large heat capacity of the ice layer.

Small reflection of sunlight from water can be neglected. In subsequent calculations, the cosine of the refraction angle is assumed constant and equal to $\mu_\mathrm{j} = 0.75$, which approximately corresponds to the zenith angle of the Sun $\theta_\mathrm{i} = 60°$. As before, the insignificant difference between the refractive indices of water and ice makes it possible to neglect the reflection and refraction of light both at the bottom of the melt pond and at the lower surface of the ice layer. To simplify the calculations, the propagation of only the direct solar radiation is considered. Of course, a 1D approach that does not take into account the size of the melt pond and its unequal depth in the central part and at the periphery is acceptable for the model problem. Perovic et al. [2021] found that the depth of melt ponds during ice melt varies little with time because it is regulated by the flow of horizontal and vertical water drainage to the ocean. As a result, some of the solar heat absorbed by the water in the pond is not transferred to the ice layer. This should be taken into account, especially for small ponds.

To calculate the transfer of solar radiation in the ice layer under the melt pond, the above suggested method can be used. Since we are not interested in the distribution of absorbed radiation over the thickness of the ice layer, it is sufficient to consider the spectral irradiation integrated over the optical thickness of the ice. However, the profile of the ice scattering parameter is not known. In addition, the solution depends on physical parameters related to solar illumination and the depth of the melt pond. The natural uncertainty of these data does not allow one to expect accurate results of detailed calculations. Under these conditions, relying on the field measurement data available in the literature is reasonable. The experimental data on geometric parameters and heat fluxes for first-year Arctic Sea ice in advanced stages of melt were given by Hudson et al. [2013].

The following approximation of the radiation power absorbed in the ice layer is used:

$$P = P_\mathrm{max} \exp(-z/d_\mathrm{ice}^*). \tag{27}$$

According to Fig. 1, at the scattering parameter $S = 2$ m^{-1} this dependence with the value $d_\mathrm{ice}^* = 0.8$ m is universal for ice layers of different thicknesses in the range $0.2 \leq d_\mathrm{ice} \leq 0.8$ m. The integration of Eq. (27) gives the following formula for the radiative flux absorbed in an ice layer:

$$q_\mathrm{ice} = q_\mathrm{ice}^\mathrm{inc}[1 - \exp(-d_\mathrm{ice}/d_\mathrm{ice}^*)]. \tag{28}$$

Of course, the dependences of $q_\mathrm{ice}(d_\mathrm{ice})$ at the real scattering parameter, which varies with the ice thickness, may differ significantly from function (28), but retrieval of profiles $S(z)$ from the available experimental data is impossible. Therefore, equation (28) should be considered an essential assumption adopted in the approximate model. The integral radiative flux absorbed in a melt pond is given by the following equation:

$$q_\mathrm{w} = \int_{\lambda_\mathrm{uv\text{-}vis}}^{\lambda_*} q_\lambda^\mathrm{w}\, d\lambda, \quad q_\lambda^\mathrm{w} = q_{\lambda,\mathrm{w}}^\mathrm{inc}\big[1 - \exp\big(-\alpha_\lambda^\mathrm{w}\, d_\mathrm{p}/\mu_\mathrm{j}\big)\big]. \tag{29}$$

It is assumed that $q_{\lambda,\mathrm{w}}^{\mathrm{inc}}$ is constant during a large part of the polar summer and its spectral dependence roughly coincides with the Planck function at temperature $T_{\mathrm{sol}} = 5800$ K. A day-averaged integral incident flux of solar radiation in the range of water and ice semitransparency $q_{\mathrm{inc}} = q_{\mathrm{w}} + q_{\mathrm{ice}}^{\mathrm{inc}}$ can be taken from [Hudson et al. 2013].

Using the approximate value of q_{ice}, one can estimate the linear rate of ice melting using the heat balance equation:

$$\rho_{\mathrm{ice}} L \dot{d}_{\mathrm{ice}} = -((1 - K_{\mathrm{loss}}) q_{\mathrm{w}} + q_{\mathrm{ice}}), \quad d_{\mathrm{ice}}(0) = d_{\mathrm{ice},0}. \tag{30}$$

The dimensionless coefficient $K_{\mathrm{loss}} < 1$ is introduced to account the heat losses with horizontal drainage of water into the ocean. The value $K_{\mathrm{loss}} = 0.2$ is used in subsequent calculations. The results of calculations presented in Fig. 6 are physically correct. In particular, two periods of the process can be distinguished: first, the ice melts faster due to the absorption of solar radiation passing through it, and then the heat is transferred to ice mainly from the melt pond. Note that the characteristic time of melting of thick ice is comparable to the duration of the warmest period of the polar summer.

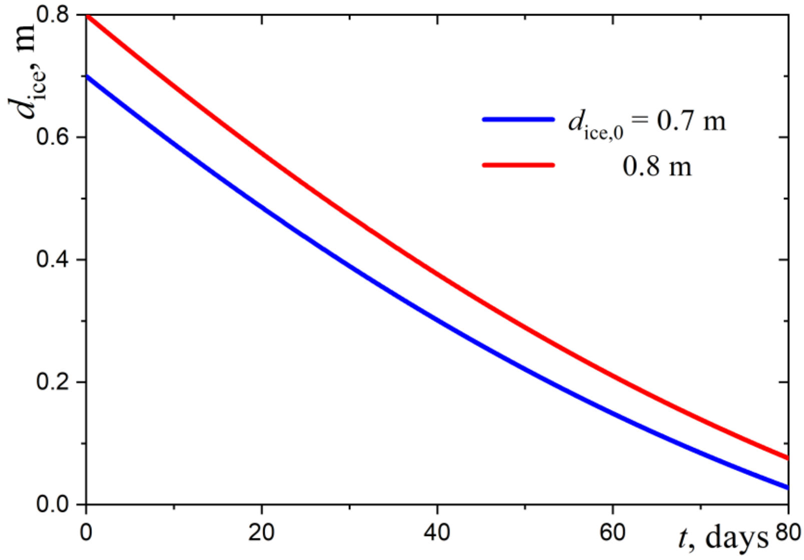

Figure 6. Time variation of ice layer thickness during the ice melting.

Of course, the suggested model can be considered only as a simple physical assessment, whereas the real process of snow and sea ice melting during the polar summer with the formation of melt ponds needs a more sophisticated analysis which is beyond the scope of the present paper. However, it is clear that a relatively simple approach to solving the radiative transfer problem may be useful in the physical modeling of one of the stages of ice melting and the complex evolution of melt ponds.

As a result of the intensive melting of sea ice during the polar summer, significant areas of the Arctic Sea may become ice-free. Most likely, this will not prevent the restoration of the ice cover during the polar winter, which would be an extremely undesirable effect of global warming. The water surface in the Arctic Sea cannot heat up above the melting point of ice due to the natural convection. Thus, the physical properties of water allow us to count on serious negative feedback and retain some optimism about the rate of global climate change accompanied by intensive seasonal melting of polar ice.

CONCLUSIONS

The present work is mainly devoted to the modeling of solar heating of ice and ice melting on the surface of lakes. The developed computational models are based on a simple but sufficiently accurate differential method for calculating radiative transfer in the scattering ice and in the snow layer that may be present on the ice surface. Typically, an analytical solution for radiative heat transfer is coupled with a numerical solution for the transient energy equation. The computational model takes into account convective heat transfer to the atmospheric air, infrared solar radiation absorbed at the illuminated surface, and radiative cooling in the mid-infrared transparency window of the cloudless atmosphere. It was shown that a thick layer of ice not covered with snow begins to melt at the ice-water interface due to solar heating of ice. The computational data are in good agreement with the field observations for Ngoring Lake in the Qinghai–Tibet Plateau.

The analysis showed a dramatic change in the process if there is a layer of snow on the ice. Even in the case of a thin snow layer less than one centimeter thick, the snow does not transmit most of the sunlight and makes ice melting impossible. At the same time, the snow itself on the ice surface does not heat up because it scatters a significant part of visible solar radiation, and the absorbed solar heat is transferred almost instantly to the relatively thick layer of ice. Snow melting can only begin when the snow layer is about 15 mm thick, and only when the snow layer is about 200 mm thick does melting become significant. If the initial thickness of the snow layer exceeds 300 mm, snow melting near the sunlit surface leads to the formation of a melt pond on the ice surface. Such melt pools are regularly observed in polar summer on the ice surface of the Arctic Sea. Abnormally strong partial melting of sea ice in the Arctic during polar summer is an extremely important process that has been intensively studied in recent years. The last part of the paper provided an estimate of ice melting under a melt pond. The results obtained are in qualitative agreement with in-situ observations.

Some results of the present work are expected to be of interest to researchers working in the same field. Perhaps more importantly, the suggested method of the approximate calculations of solar heating and melting of lake or sea ice on the water surface, taking into account possible snow cover on the ice, can be used as the basis of computational models for a variety of applied problems in geophysics.

ACKNOWLEDGEMENTS

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Biography

Leonid A. Dombrovsky is a highly regarded expert in the field of radiative transfer, specializing in the study of various scattering media as applied to power engineering, geophysics, and biomedicine. Currently holding the position of Chief Researcher at the Joint Institute for High Temperatures in Moscow, Russia, he has made significant contributions to the field in his career. In 1974, Leonid Dombrovsky earned his PhD from MIPT, followed by the completion of his Doctor of Science degree in 1990. Throughout his extensive research journey, Prof. Dombrovsky has published over 300 research papers, as well as authored several books and book chapters. Of particular note are his monographs on radiative and combined heat transfer in scattering media, which were published by Begell House (New York) in 1996 and 2010. These monographs are extensively referenced by researchers worldwide. In recognition of his achievements in the science and art of heat and mass transfer, Prof. Dombrovsky was awarded the prestigious A.V. Luikov Medal in 2016, followed by the William Begell Medal in 2018. The cooperation of Leonid Dombrovsky with his colleagues from Australia, Germany, France, Israel, Russia, Sweden, Switzerland, the UK, and the USA over the past 28 years has played a crucial role in enhancing the overall expertise and diversity of the physical models developed. As a result, advances have been achieved in a wide range of fields, encompassing heat transfer in rocket engines and solar thermochemical reactors, thermal processes in industrial nuclear reactors, microwave emission of foam on the ocean surface, radar remote sensing of atmospheric clouds, solar heating and melting of snow and ice, shielding of thermal radiation by evaporating or sublimating droplet and solid particle clouds, stabilization and control of levitating droplet clusters, airborne spreading of viruses, and infrared thermal treatment of human tumors.